Lecture 8

Frequency Responses of a System

Peter Cheung
Department of Electrical & Electronic Engineering
Imperial College London

URL: www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
E-mail: p.cheung@imperial.ac.uk

Transfer Function H(s) vs Frequency Response H(jω)

Laplace Transform

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^\infty x(t)e^{-st}dt$$

$s = j\omega$

Fourier Transform

$$\mathcal{F}\{x(t)\} = X(j\omega) = \int_0^\infty x(t)e^{-j\omega t}dt$$

Transfer Function

$$H(s)$$

$s = j\omega$

Frequency Response

$$H(j\omega)$$

- H(jω) is often expressed in polar form:

Frequency Response

$$H(s)\Big|_{s=j\omega} = H(j\omega) = |H(j\omega)|\, e^{j\angle H(j\omega)}$$

Amplitude Response

Phase Response

$$\cos\omega t \rightarrow \boxed{H(j\omega)} \rightarrow |H(j\omega)|\cos[\omega t + \angle H(j\omega)]$$

L4.8 p423

Frequency Response Example (1)

- Find the frequency response of a system with transfer function:

$$H(s) = \frac{s+0.1}{s+5}$$

- Then find the amplitude and phase response $y(t)$ for inputs:

  (i) $x(t)=\cos 2t$ and (ii) $x(t)=\cos(10t-50°)$

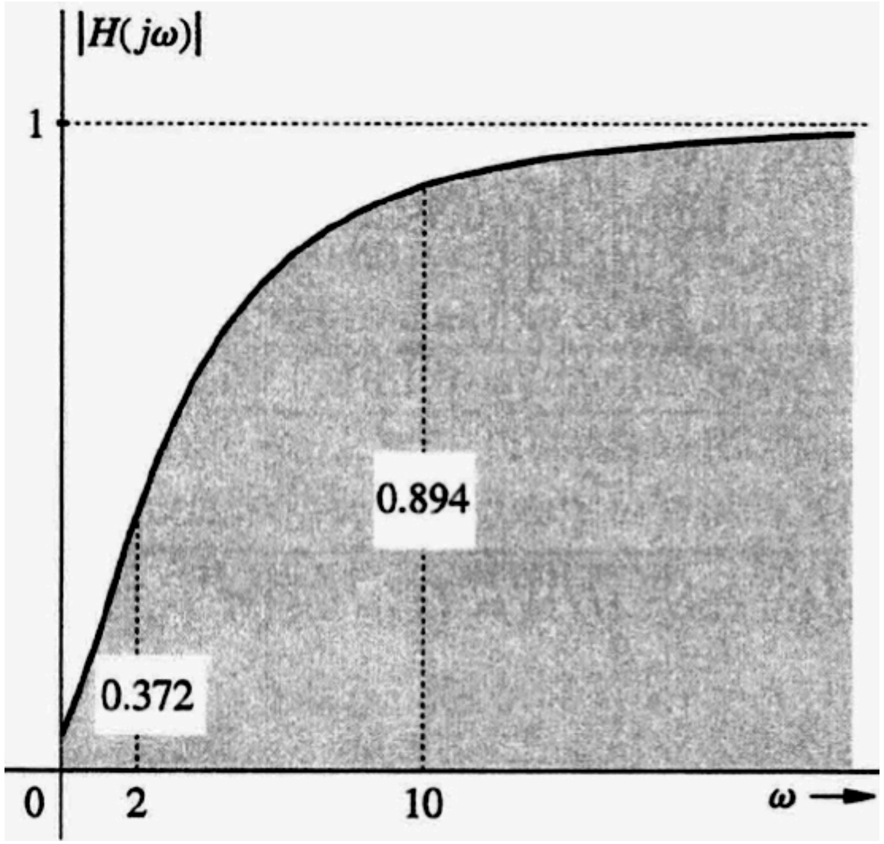
- Substitute $s=j\omega$

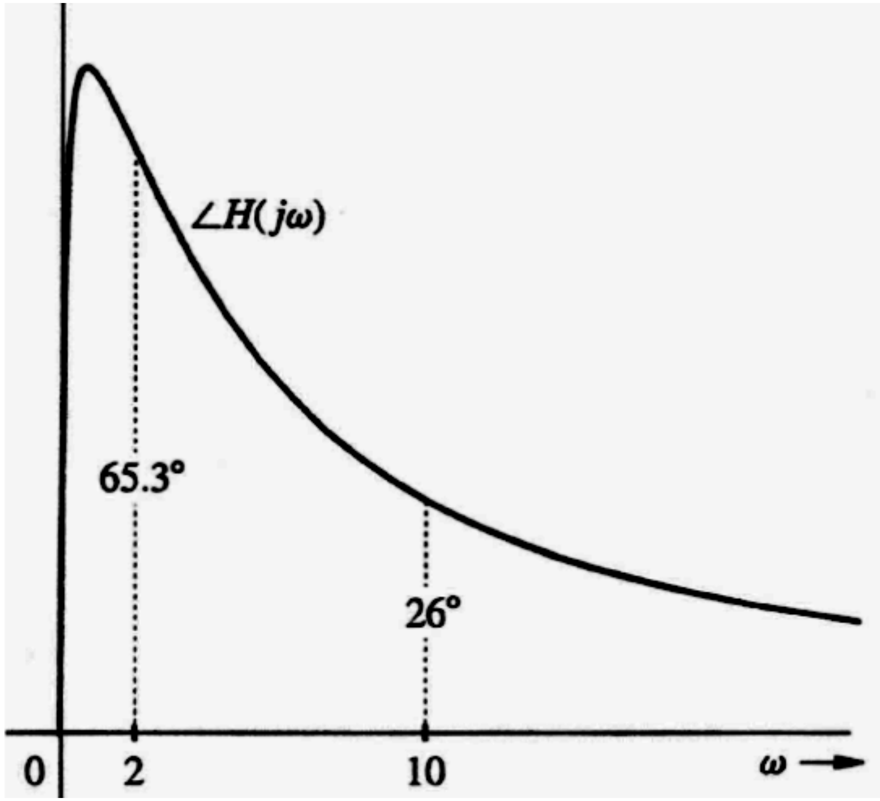
$$H(j\omega) = \frac{j\omega+0.1}{j\omega+5}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2+0.01}}{\sqrt{\omega^2+25}} \quad \text{and} \quad \angle H(j\omega) = \Phi(j\omega) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

Frequency Response Example (2)

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 0.01}}{\sqrt{\omega^2 + 25}} \qquad \Phi(j\omega) = \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$





for input $x(t)=cos2t$ and $x(t)=cos(10t\text{-}50°)$

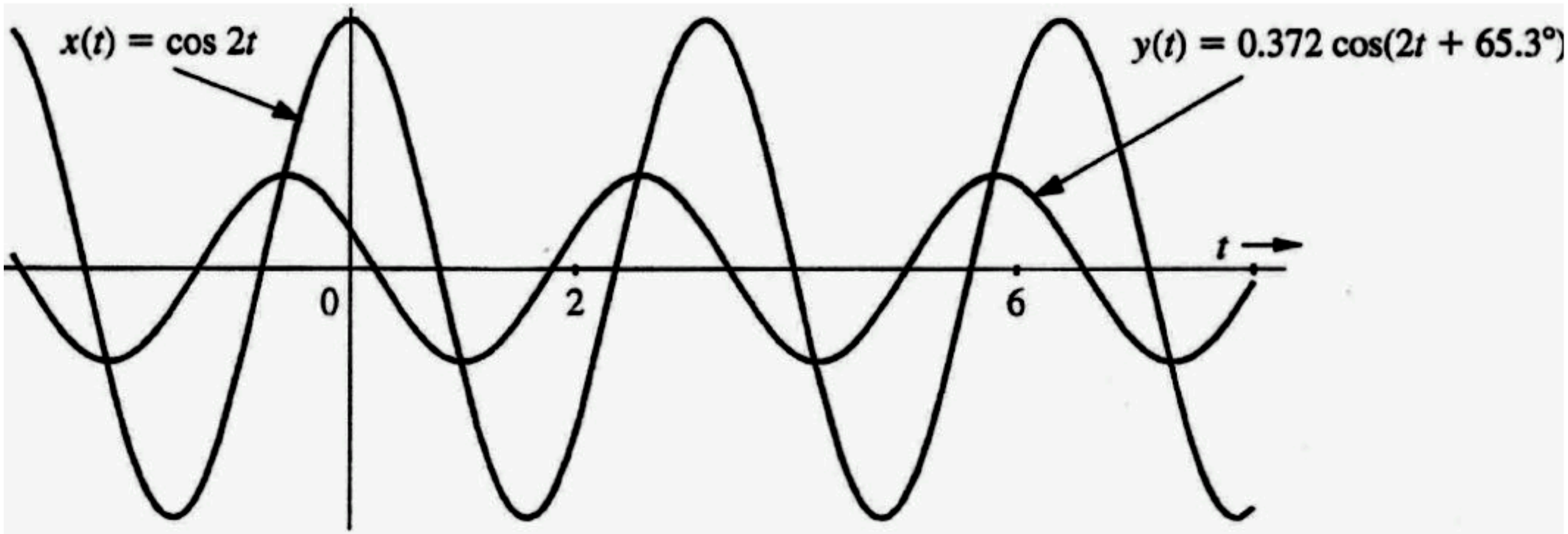
Frequency Response Example (3)

- For input $x(t)=cos2t$, we have:

$$|H(j2)| = \frac{\sqrt{2^2+0.01}}{\sqrt{2^2+25}} = 0.372 \qquad \Phi(j2) = \tan^{-1}\left(\frac{2}{0.1}\right) - \tan^{-1}\left(\frac{2}{5}\right) = 65.3^\circ$$

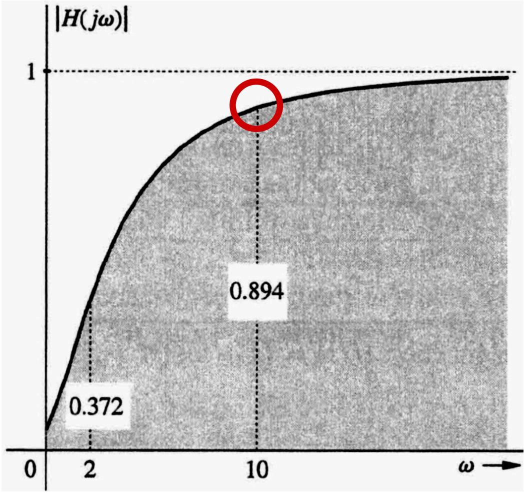
- Therefore

$$y(t) = 0.372\cos(2t + 65.3^\circ)$$



Frequency Response Example (4)

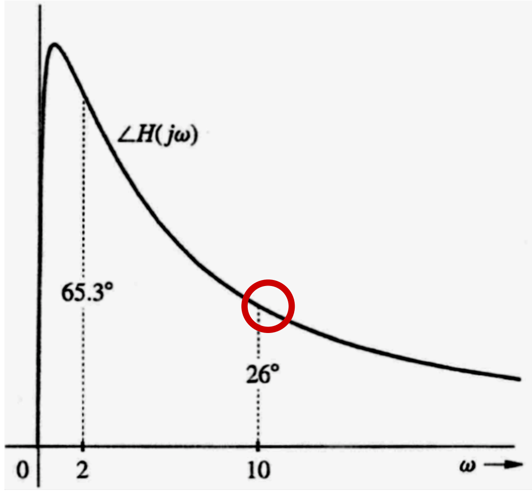
- For input $x(t)=cos(10t\text{-}50°)$, we will use the amplitude and phase response curves directly:



$$|H(j10)| = 0.894$$

$$\Phi(j10) = \angle H(j10) = 26^\circ$$

- Therefore



$$y(t) = 0.894\cos(10t - 50^\circ + 26^\circ) = 0.894\cos(10t + 24^\circ)$$

Frequency Response of delay of T sec

- H(s) of an ideal T sec delay is:

$$H(s) = e^{-sT} \qquad \text{(Time-shifting property)}$$

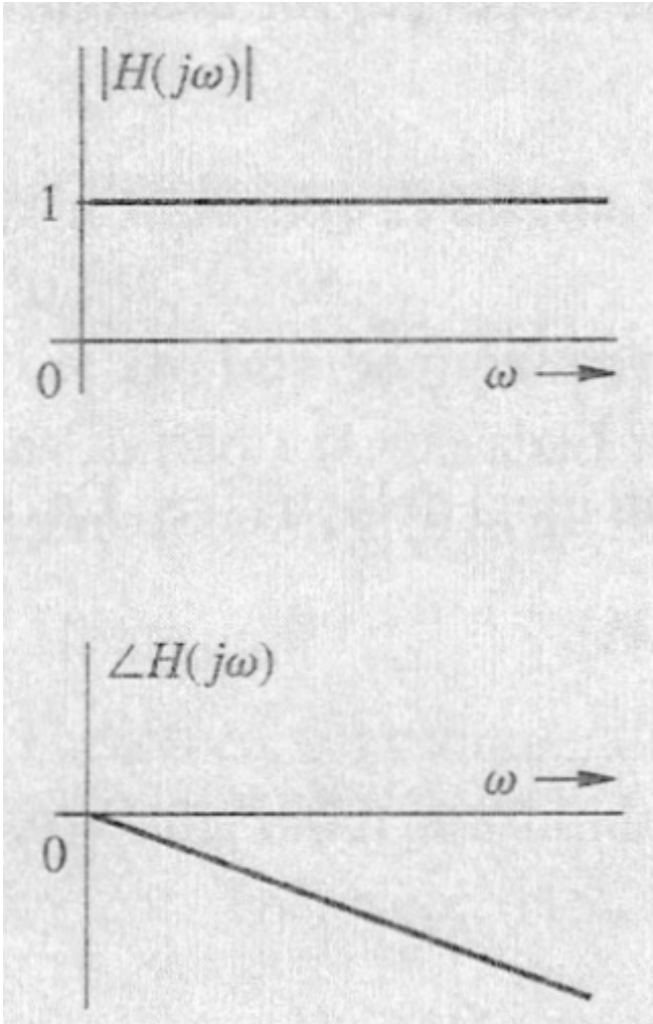
- Therefore

$$|H(j\omega)| = \left|e^{-j\omega T}\right| = 1 \quad \text{and} \quad \Phi(j\omega) = -\omega T$$

- That is, delaying a signal by T has **no effect** on its amplitude.
- It results in a linear phase shift (with frequency), and a gradient of –T.
- The quantity:

$$-\frac{d\Phi(\omega)}{d\omega} = \tau_g = \mathrm{T}$$

is known as **Group Delay**.



Frequency Response of an ideal differentiator

- H(s) of an ideal differentiator is:

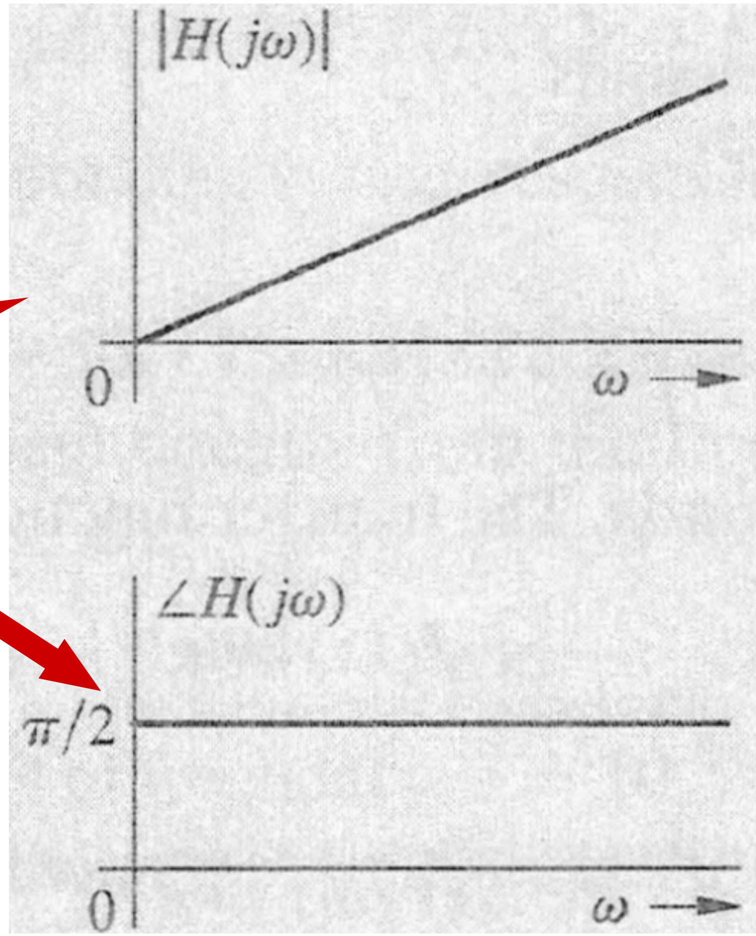
$$H(s) = s \quad \text{and} \quad H(j\omega) = j\omega = \omega e^{j\pi/2}$$

- Therefore

$$|H(j\omega)| = \omega \quad \text{and} \quad \angle H(j\omega) = \frac{\pi}{2}$$

- This agrees with:

$$\frac{d}{dt}(\cos\omega t) = -\omega\sin\omega t = \omega\cos(\omega t + \pi/2)$$



- That's why differentiator is **not** a nice component to work with – it **amplifies high frequency** component (i.e. noise!).

Frequency Response of an ideal integrator

- H(s) of an ideal integrator is:

$$H(s) = \frac{1}{s} \quad \text{and} \quad H(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega} = \frac{1}{\omega} e^{-j\pi/2}$$

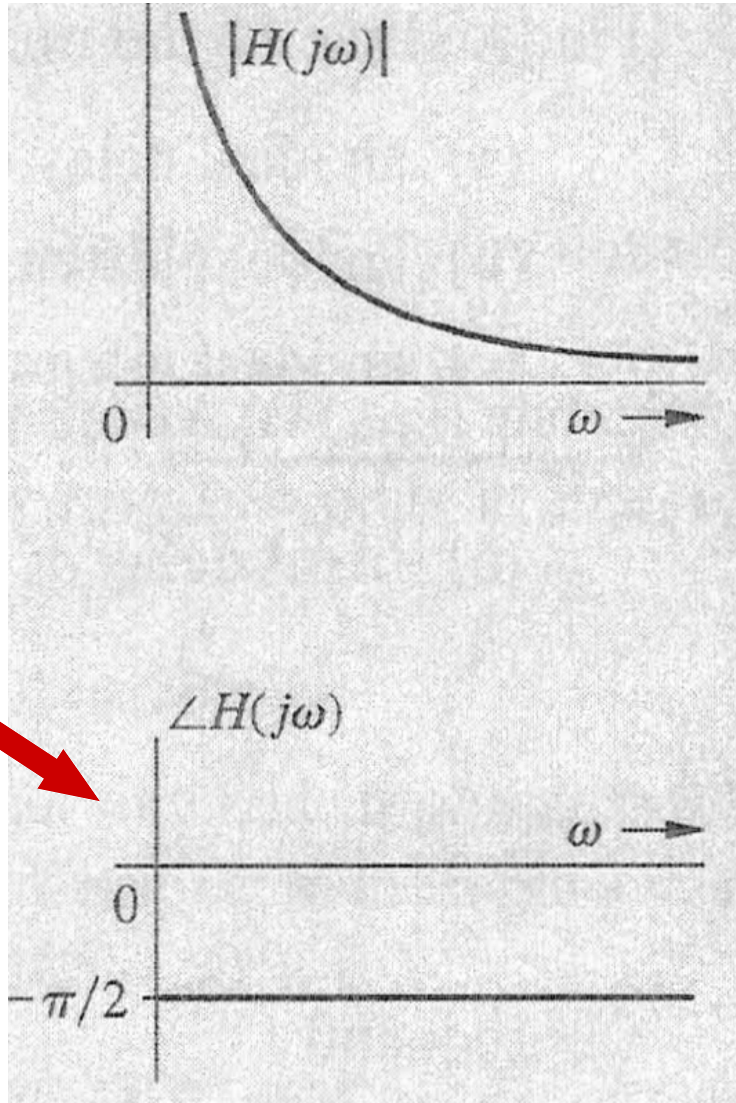
- Therefore

$$|H(j\omega)| = \frac{1}{\omega} \quad \text{and} \quad \angle H(j\omega) = -\frac{\pi}{2}$$

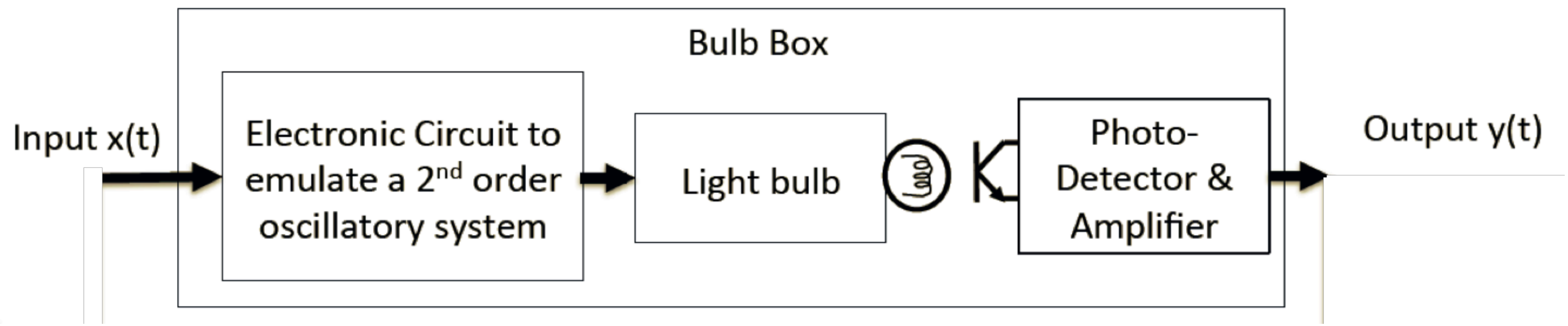
- This agrees with:

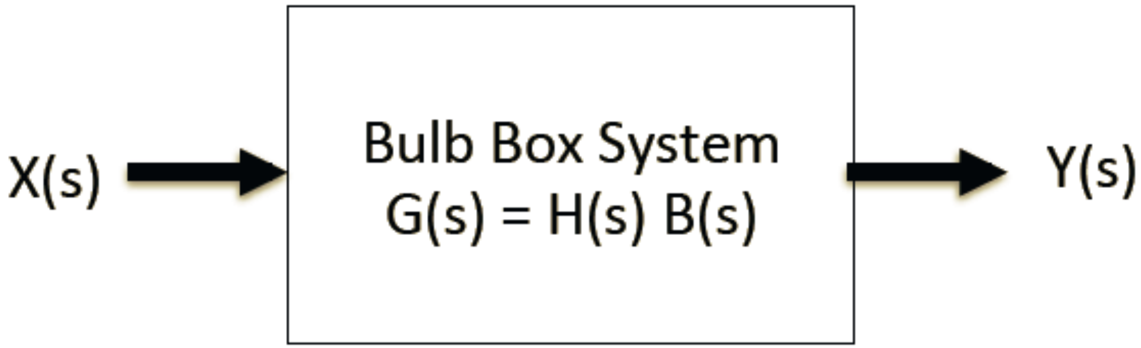
$$\int \cos \omega t \, dt = \frac{1}{\omega} \sin \omega t = \frac{1}{\omega} \cos(\omega t - \pi / 2)$$

- That's why integrator is a nice component to work with – it suppresses high frequency component (i.e. noise!).



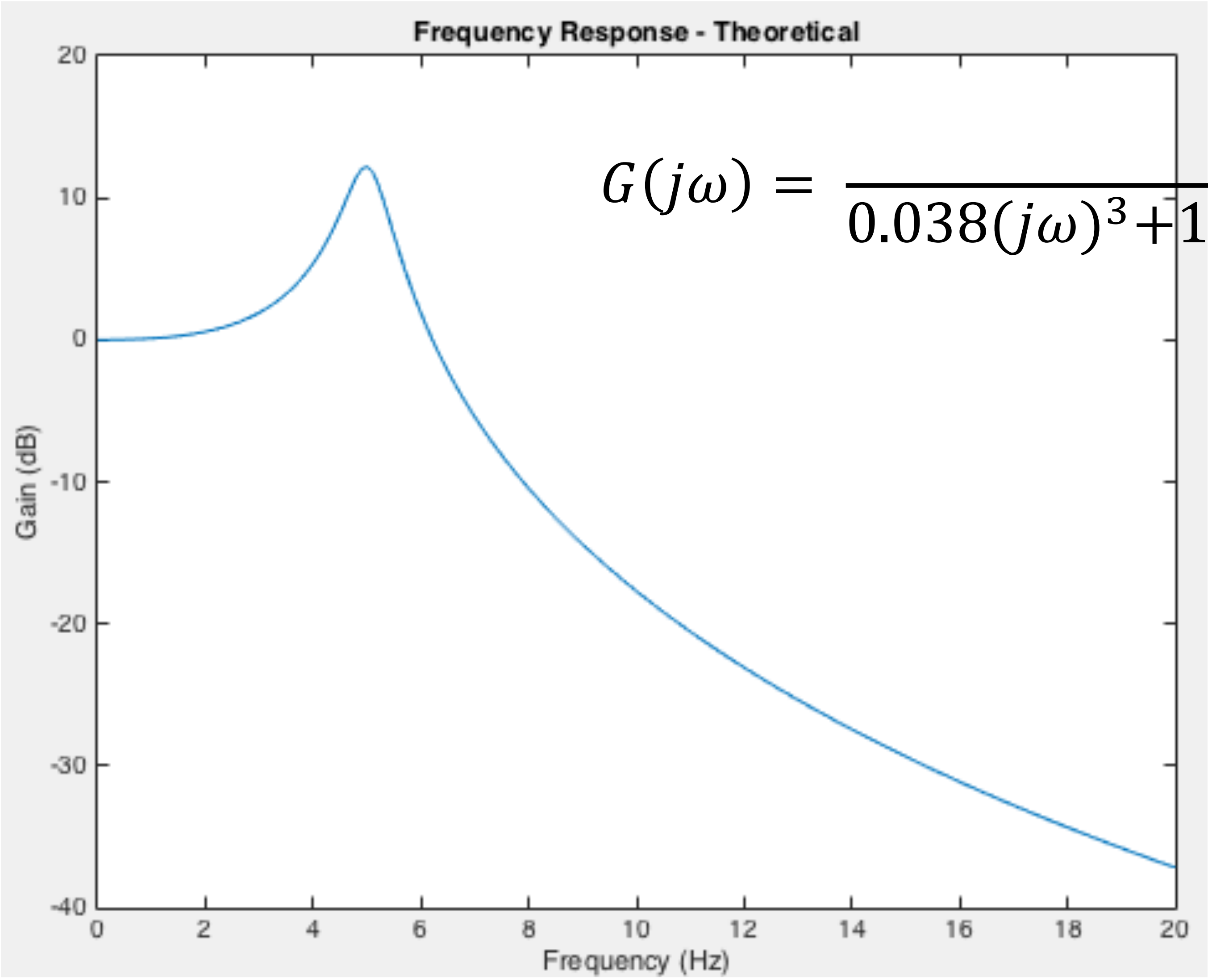
Frequency Response of Bulb Box





$$G(s) = \frac{1000}{0.038s^3 + 1.19s^2 + 43s + 1000}$$

Theoretical Frequency Response of Bulb Box



$$G(j\omega) = \frac{1000}{0.038(j\omega)^3 + 1.19(j\omega)^2 + 43j\omega + 1000}$$